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Determining the Efficiency of Fuzzy Logic EOQ Inventory Model with Varying Demand in Comparison with Lagrangian and Kuhn-Tucker Method Through Sensitivity Analysis

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Abstract

This paper considers an EOQ inventory model with varying demand and holding costs. It suggests minimizing the total cost in a fuzzy related environment. The optimal policy for the nonlinear problem is determined by both Lagrangian and Kuhn-tucker methods and compared with varying price-dependent coefficient. All the input parameters related to inventory are fuzzified by using trapezoidal numbers. In the end, a numerical example discussed with sensitivity analysis is done to justify the solution procedure. This paper primarily focuses on the aspect of Economic Order Quantity (EOQ) for variable demand using Lagrangian, Kuhn-Tucker and fuzzy logic analysis. Comparative analysis of there methods are evaluated in this paper and the results showed the efficiency of fuzzy logic over the conventional methods. Here in this research trapezoidal fuzzy numbers are incorporated to study the price dependent coefficients with variable demand and unit purchase cost over variable demand. The results are very close to the crisp output. Sensitivity analysis also done to validate the model.

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Introduction

In today's competitive scenario, organizations face immense challenges for meeting the transitional consumer's demand, and maintaining the inventory plays a major role. Indulging the betterment of various promotional activities that yield a reputation for the concern. The classical economic order quantity by Harris [1] and Wilson [2] was developed for problems where demand remains constant. In recent study of inexplicit demand rates by Chen [3] and few more resulting in the time-dependent demand rates and varying holding cost. In 1965, Zadeh [4] introduced the concept of fuzzy that carries vagueness or unclear in sense. Hence fuzzy sets came into prominence in describing the vagueness and uncertainty that impressed many researchers. In this paper, the total cost has been taken and the proposed model economic order quantity (EOQ) and the input parameters such as ordering cost, purchase cost, order size, holding cost, unit purchase cost, and unit selling price are fuzzified using trapezoidal numbers. The optimization is carried out by both Lagrangian and Kuhn-Tucker methods and graded mean integration for defuzzification of the total cost.

In the early 1990's the Economic order quantity (EOQ) and Economic Production Quantity (EPQ) played a vital role in the operations management area, contrastingly it failed in meeting with the real-world challenges. Since these models assumed that the items received or produced are of a perfect quality which is highly challenging. In inventory management, EOQ plays a vital role in minimizing holding and ordering costs. Ford W. Harris (1915) developed the EOQ model but it was further extended and extensively applied by R.H. Wilson. Wang[5] examined an EOQ model where a proportion of defective items were represented as fuzzy variables. In the year 2000, Salameh et al [6] came up with a classical EOQ that assumed a random proportion of defective items, and the recognized imperfect items are sold in a single batch at an economical rate. Chen [3] in the year 2003, came up with an EOQ with a random demand that minimized the total cost. Firms to cope up with the current scenarios, have to adapt the diversity among inventory models due to the advancement of management strategies and varying production levels.

Briefly, the input parameters of inventory models are often taken as crisp values due to variability in nature.H.J. Zimmerman [7] studied the fuzziness in operational research. In a classical EOQ, Park [8] fuzzified the decision variables ordering cost and holding cost into trapezoidal fuzzy numbers that gave rise to study the fuzzy EOQ models, solving non-linear programming to obtain the optimal solution for economic order quantity. In 2002, Hsieh [9] initiated two production inventory models that applied the graded -mean representation method for the defuzzification process. In traditional inventory models when minimizing the annual costs, the demand rate was always assumed to be independent. Despite the promotional setups and the deterioration items, a wavering demand arises. In 2003, Sujit [10] came up with an inventory model that involves a fuzzy demand rate fuzzy deterioration rate. In 2013, Dutta and Pavan Kumar [11] proposed an inventory model without shortfall with fuzziness in demand, holding cost, and ordering cost. The study of inexplicit demand rates was doneby Chan [12] and few more resulting in the time-dependent demand rates and varying holding cost. A detailed study of Lagrangian method to solve the non-linear programming of the total cost was done by Kalaiarasi et al[13]. The Kuhn-Tucker optimization technique was for the NPP(non-linear programming) problem was carried out by Kalaiarasi et al[14].

Considering these inputs, the non-linear programming problem is solved for total cost function using Lagrangian and Kuhn-Tucker method and concluded with a sensitivity analysis, which exhibits the variations between the fuzzy and crisp values.

Preliminaries

Definition 1:

A fuzzy set Ã defined on R [∞,-∞], if the membership function of Ã is defined by

$$\mu_{\tilde{A}}(x) = \begin{cases} 1, x \in A \\ 0, x \notin A \end{cases}$$

Definition 2:

A trapezoidal fuzzy number Ã=(a, b, c, d) with a membership function μÃ is defined by

$$y = m(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{m-a}, & a < x \le m \\ \frac{b-x}{b-m}, & m < x < b \\ 0, & x \ge b \end{cases}$$

The Fuzzy Arithmetical Operations

Function principle [12] is proposed to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. Defining some fundamental fuzzy arithmetical operations under function principle as follows

Suppose $\tilde{A}=(x_1, x_2, x_3, x_4)$ and $\tilde{B} = (y_1, y_2, y_3, y_4)$ be two trapezoidal fuzzy numbers. Then

The addition of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} \oplus \bar{B} = (x_1+y_1, x_2+y_2, x_3+y_3, X_4+y_4)$

Where $x_1, x_2, x_3, x_4, y_1, y_2, y_3, y_4$ are any real numbers.

(2) The multiplication of $\tilde{A}$ and $\tilde{B}$ is

$\tilde{A} \oplus \tilde{B} = (c_1, c_2, c_3, c_4)$

Where $Z_1=\{x_1y_1, x_1y_4, x_4y_1, x_4y_4\}$, $Z_2 = \; =\{x_2y_2, x_2y_3, x_3y_2, x_3y_4\}$, $C_1 = \min Z_1$, $C_2 = \min Z_1$, $C_1 = \max Z_2$, $C_1 = \max Z_2$.

If $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and y_4 are all zero positive real numbers then

$\tilde{A} \otimes \tilde{B} = (x_1y_1, x_2y_2, x_3y_3, x_4y_4)$.

(3) The subtraction $\tilde{A}$ and $\tilde{B}$ of is

$$\tilde{A} \ominus \tilde{B} = (x_1 - y_4,\ x_2 - y_3,\ x_3 - y_2,\ x_4 - y_1).$$

Where $-\tilde{B} = (-y_4, -y_3, -y_2, -y_1)$, also $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and y_4 are any real numbers.

(4) The division of $\tilde{A}$ and $\tilde{B}$ is

$$\tilde{A} \oslash \tilde{B} = \left(\frac{x_1}{y_4}, \frac{x_2}{y_3}, \frac{x_3}{y_2}, \frac{x_4}{y_1}\right).$$

$$\frac{1}{\tilde{B}} = \tilde{B}^{-1} = \left(\frac{1}{y_4}, \frac{1}{y_3}, \frac{1}{y_2}, \frac{1}{y_1}\right)$$

where y_1, y_2, y_3 and y_4 are positive real numbers. Also $x_1, x_2, x_3, x_4, y_1, y_2, y_3$ and $y_4$are nonzero positive numbers.

(5) For any $\propto \in R$

(a) If $\propto \ge 0$, then $\propto \otimes \tilde{A} = (\propto a_1, \propto a_2, \propto a_3, \propto a_4)$.

(b) If $\propto < 0$, then $\propto \otimes \tilde{A} = (\propto a_4, \propto a_3, \propto a_2, \propto a_1)$.

Extension of the Lagrangian Method.

Solving a nonlinear programming problem by obtaining the optimum solution was discussed by Taha[13] with equality constraints, also by solving those inequality constraints using Lagrangian method.

Suppose if the problem is given as

Minimize y = f(x)

Sub to $g_i(x) \ge 0$, i = 1, 2, · · ·, m.

The constraints are non-negative say $x \ge 0$ if included in the m constraints. Then the procedure of the extension of the Lagrangian method will involve the following steps.

Step 1:

Solve the unconstrained problem

Min y = f(x)

If the resulting optimum satisfies all the constraints, then stop since all the constraints are inessential. Or else set K = 1 and move to step 2.

Step 2:

Activate any K constraints (i.e., convert them into equalities) and optimize f(x) subject to the K active constraints by the Lagrangian method. If the resulting solution is feasible with respect to the remaining constraints, the steps have to be repeated. If all sets of active constraints taken K at a time are considered without confront a feasible solution, go to step 3.

Step 3:

If K = m, stop; there's no feasible solution.

Otherwise set K = K + 1 and go to step 2.

Graded Mean Integration Representation Method

Graded mean Integration Representation Method was introduced by Hsieh et al [14] based on the integral value of graded mean h-level of generalized fuzzy number for the defuzzification of generalized fuzzy number. First, a generalized fuzzy number is defined as follows: $\tilde{A}$-=$(\alpha_1, \alpha_2, \alpha_3, \alpha_4,)_{LR}$. By graded mean integration are the inverses of L and R are L^{-1} and R^{-1} respectively. The graded mean h-level value of the generalized fuzzy number $\tilde{A}$-=$(\alpha_1, \alpha_2, \alpha_3, \alpha_4,)_{LR}$ is given by $h/2[L^{-1}(h)+R^{-1}(h)]$. Then the graded mean integration representation of $P(\tilde{A})$ with grade then

$$P(\tilde{A}) = \frac{\int_0^{\omega_A} \frac{h}{2}\left[L^{-1}(h) + R^{-1}(h)\right] dh}{\int_0^{\omega_A} h \, dh}$$

Where $0 < h \leq \omega_A$ and $0 < \omega_A \leq 1$.

In this paper trapezoidal fuzzy numbers is used as fuzzy parameters for the production inventory model. Let $\tilde{B} = (b_1, b_2, b_3, b_4)$ Then the graded mean integration representation is given by the formula as

$$P(\tilde{B}) = \frac{\int_0^1 \frac{h}{2}\left[(b_1 + b_4) + h(b_2 - b_1 - b_4 + b_3)\right] dh}{\int_0^1 h \, dh}$$

$$= \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

Inventory Model For Eoq

The input parameters for the corresponding model are

K - Ordering cost

a - constant demand rate coefficient

b - price-dependent demand rate coefficient

P - selling price

Q - Order size

c - unit purchasing cost

g - constant holding cost coefficient

Let's consider the total cost [15] and modify in a fuzzy environment

The total cost per cycle is given by

$$TC(Q,P) = \frac{K(a - bP)}{Q} + c(a - bP) + \frac{gcQ}{2}$$

Partially differentiating w.r.t Q,

$$\frac{\partial T}{\partial Q} = \frac{K(a - bP)}{Q^2} + \frac{gc}{2}$$

Equating $\frac{\partial T}{\partial Q} = 0$ the economic order quantity in crisp values obtained as

$$Q = \sqrt{\frac{2K(a - bP)}{gc}}$$

Inventory Model For Fuzzy Order Quantity

By using trapezoidal numbers, fuzzified input parameters are as follows

Suppose $\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$, $\tilde{K} = (K_1, K_2, K_3, K_4)$, $\tilde{P} = (P_1, P_2, P_3, P_4)$, $\tilde{c} = (c_1, c_2, c_3, c_4)$, $\tilde{a} = (a_1, a_2, a_3, a_4)$,

$\tilde{b} = (b_1, b_2, b_3, b_4)$ and $\tilde{g} = (g_1, g_2, g_3, g_4)$

The optimal order quantity

$$= \frac{1}{6}\left[\left(\frac{K_1(a_1 - b_1P_1)}{Q} + c_1(a_1 - b_1P_1) + \frac{g_1c_1Q}{2}\right) + 2\left(\frac{K_2(a_2 - b_2P_2)}{Q} + c_2(a_2 - b_2P_2) + \frac{g_2c_2Q}{2}\right) + 2\left(\frac{K_3(a_3 - b_3P_3)}{Q} + c_3(a_3 - b_3P_3) + \frac{g_3c_3Q}{2}\right) + \left(\frac{K_4(a_4 - b_4P_4)}{Q} + c_4(a_4 - b_4P_4) + \frac{g_4c_4Q}{2}\right)\right]$$

Partially differentiating w.r.t 'Q' and equating to zero,

$$= \frac{1}{6}\left[-\frac{K_1(a_1 - b_1P_1)}{Q^2} + \frac{g_1c_1}{2} - \frac{2K_2(a_2 - b_2P_2)}{Q^2} + \frac{g_2c_2}{2} - \frac{2K_3(a_3 - b_3P_3)}{Q^2} + \frac{g_3c_3}{2} - \frac{K_4(a_4 - b_4P_4)}{Q^2} + \frac{g_4c_4}{2}\right]$$

Hence the optimal economic order quantity for crisp values is derived,

$$Q = \sqrt{\frac{2K_1(a_1 - b_1P_1) + 4K_2(a_2 - b_2P_2) + 4K_3(a_3 - b_3P_3) + 2K_4(a_4 - b_4P_4)}{g_1c_1 + g_2c_2 + g_3c_3 + g_4c_4}}$$

Applying the graded mean representation

$$= \frac{1}{6}\left[\left(\frac{K_1(a_1 - b_1P_1)}{Q_4} + c_1(a_1 - b_1P_1) + \frac{g_1c_1Q_1}{2}\right) + 2\left(\frac{K_2(a_2 - b_2P_2)}{Q_3} + c_2(a_2 - b_2P_2) + \frac{g_2c_2Q_2}{2}\right) + 2\left(\frac{K_3(a_3 - b_3P_3)}{Q_2} + c_3(a_3 - b_3P_3) + \frac{g_3c_3Q_3}{2}\right) + \left(\frac{K_4(a_4 - b_4P_4)}{Q_1} + c_4(a_4 - b_4P_4) + \frac{g_4c_4Q_4}{2}\right)\right]$$

Now partially differentiating w.r.t Q_1, Q_2, Q_3, Q_4 and equating to zero,

$$Q_1 = \sqrt{\frac{2K_4(a_4 - b_4P_4)}{g_1c_1}}$$

$$Q_2 = \sqrt{\frac{2K_3(a_3 - b_3P_3)}{g_2c_2}}$$

$$Q_3 = \sqrt{\frac{2K_2(a_2 - b_2P_2)}{g_3c_3}}$$

$$Q_4 = \sqrt{\frac{2K_1(a_1 - b_1P_1)}{g_4c_4}}$$

The above derived results depict that $Q_1 > Q_2 > Q_3 > Q_4$ failing to satisfy the constraints $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. So, converting the inequality constraint $Q_2 - Q_1 \ge 0$ into equality constraint $Q_2 - Q_1 = 0$ Optimizing P(TC(Q,P)) subject to $Q_2 - Q_1 = 0$ by Lagrangian method.

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = P(TC(Q,P)) - \lambda(Q_2 - Q_1)$$

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = \frac{1}{6}\left[\left(\frac{K_1(a_1 - b_1P_1)}{Q_4} + c_1(a_1 - b_1P_1) + \frac{g_1c_1Q_1}{2}\right) + 2\left(\frac{K_2(a_2 - b_2P_2)}{Q_3} + c_2(a_2 - b_2P_2) + \frac{g_2c_2Q_2}{2}\right) + 2\left(\frac{K_3(a_3 - b_3P_3)}{Q_2} + c_3(a_3 - b_3P_3) + \frac{g_3c_3Q_3}{2}\right) + \left(\frac{K_4(a_4 - b_4P_4)}{Q_1} + c_4(a_4 - b_4P_4) + \frac{g_4c_4Q_4}{2}\right)\right] - \lambda(Q_2 - Q_1)$$

$$\frac{\partial L}{\partial Q_1} == \frac{1}{6}\left[\frac{g_1c_1}{2} - \frac{K_4(a_4 - b_4P_4)}{Q_1^2}\right] + \lambda = 0 \qquad (1)$$

$$\frac{\partial L}{\partial Q_2} == \frac{1}{6}\left[g_2c_2 - \frac{2K_3(a_3 - b_3P_3)}{Q_2^2}\right] - \lambda = 0 \qquad (2)$$

$$\frac{\partial L}{\partial Q_2} == \frac{1}{6}\left[g_3c_3 - \frac{2K_2(a_2 - b_2P_2)}{Q_3^2}\right] = 0$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{6}\left[\frac{g_4c_4}{2} - \frac{K_1(a_1 - b_1P_1)}{Q_4^2}\right] = 0$$

$$\frac{\partial P}{\partial \lambda} = -(Q_2 - Q_1)$$

From equations (1) and (2) the results are,

$$Q_1 = Q_2 = \sqrt{\frac{2K_4(a_4 - b_4P_4) + 2K_3(a_3 - b_3P_3)}{g_1c_1 + g_2c_2}}$$

$$Q_3 = \sqrt{\frac{2K_2(a_2 - b_2P_2)}{g_3c_3}}$$

$$Q_4 = \sqrt{\frac{2K_1(a_1 - b_1P_1)}{g_4c_4}}.$$

Since $Q_3 > Q_4$ which does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Now converting the inequality constraints $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$ into equality constraints $Q_2 - Q_1 = 0$ and $Q_3 - Q_2 = 0$. Optimizing,

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = P\big(TC(Q, P)\big) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2).$$

$$\begin{aligned} L(Q_1, Q_2, Q_3, Q_4, \lambda_1, \lambda_2) &= \frac{1}{6}\Bigg[\left(\frac{K_1(a_1 - b_1P_1)}{Q_4} + c_1(a_1 - b_1P_1) + \frac{g_1c_1Q_1}{2}\right) \\ &+ 2\left(\frac{K_2(a_2 - b_2P_2)}{Q_3} + c_2(a_2 - b_2P_2) + \frac{g_2c_2Q_2}{2}\right) \\ &+ 2\left(\frac{K_3(a_3 - b_3P_3)}{Q_2} + c_3(a_3 - b_3P_3) + \frac{g_3c_3Q_3}{2}\right) \\ &+ \left(\frac{K_4(a_4 - b_4P_4)}{Q_1} + c_4(a_4 - b_4P_4) + \frac{g_4c_4Q_4}{2}\right)\Bigg] - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2). \end{aligned}$$

$$\frac{\partial P}{\partial \lambda_1} = -(Q_2 - Q_1), \quad \frac{\partial P}{\partial \lambda_2} = -(Q_3 - Q_2)$$

$$\frac{\partial L}{\partial Q_1} == \frac{1}{6}\left[\frac{g_1c_1}{2} - \frac{K_4(a_4 - b_4P_4)}{Q_1^2}\right] + \lambda_1 = 0 \qquad (1')$$

$$\frac{\partial L}{\partial Q_2} == \frac{1}{6}\left[g_2c_2 - \frac{2K_3(a_3 - b_3P_3)}{Q_2^2}\right] - \lambda_1 + \lambda_2 = 0 \qquad (2')$$

$$\frac{\partial L}{\partial Q_3} = \frac{1}{6}\left[g_3c_3 - \frac{2K_2(a_2 - b_2P_2)}{Q_3^2}\right] - \lambda_2 = 0 \qquad (3')$$

$$\frac{\partial L}{\partial Q_4} = \frac{1}{6}\left[\frac{g_4c_4}{2} - \frac{K_1(a_1 - b_1P_1)}{Q_4^2}\right] = 0 \qquad (4')$$

From equations (1′) , (2′) and (3′),

$$Q_1 = Q_2 = Q_3 = \sqrt{\frac{2K_4(a_4 - b_4P_4) + 2K_3(a_3 - b_3P_3) + 2K_2(a_2 - b_2P_2)}{g_1c_1 + g_2c_2 + g_3c_3}}$$

$$Q_4 = \sqrt{\frac{2K_1(a_1 - b_1P_1)}{g_4c_4}}.$$

In the above-mentioned results Since $Q_1 > Q_4$ which does not satisfy the constraint $0 \le Q_1 \le Q_2 \le Q_3 \le Q_4$. Converting the inequality constraints $Q_2 - Q_1 \ge 0$, $Q_3 - Q_2 \ge 0$ and $Q_4 - Q_3 \ge 0$ into equality constraints $Q_2 - Q_1 = 0$, $Q_3 - Q_2 = 0$ and $Q_4 - Q_3 = 0$. Optimizing,

$$L(Q_1, Q_2, Q_3, Q_4, \lambda) = P\big(TC(Q,P)\big) - \lambda_1(Q_2 - Q_1) - \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3) \quad (5')$$

After Partially differentiation (Appendix)

$$= \sqrt{\frac{2K_4(a_4 - b_4P_4) + 2K_3(a_3 - b_3P_3) + 2K_2(a_2 - b_2P_2) + 2K_1(a_1 - b_1P_1)}{g_1c_1 + g_2c_2 + g_3c_3 + g_4c_4}}$$

$$\tilde{Q} = (Q_1, Q_2, Q_3, Q_4)$$

satisfies the required inequality constraints.

Kuhn-Tucker Optimization Method

By applying Kuhn Tucker conditions, the total cost is minimized by finding the solution of Q_1, Q_2, Q_3, Q_4 with Q_1, Q_2, Q_3, Q_4

$$\nabla f(P(TC(Q,P))) - \lambda \nabla g(Q_i) = 0$$

$$\lambda_i g_i(Q_i) = 0$$

$$g_i(Q_i) \ge 0$$

The above conditions simplify to the following $\boldsymbol{\lambda}_1$, $\boldsymbol{\lambda}_2$, $\boldsymbol{\lambda}_3$, $\boldsymbol{\lambda}_4$.

$$\nabla f(P(TC(Q,P))) - \lambda \nabla g(Q_i) = 0$$

$$\Rightarrow \frac{K_1(a_1 - b_1P_1)}{6Q_4} + \frac{c_1(a_1 - b_1P_1)}{6} + \frac{g_1c_1Q_1}{12} + \frac{2K_2(a_2 - b_2P_2)}{6Q_3} + \frac{2c_2(a_2 - b_2P_2)}{6} + \frac{g_2c_2Q_2}{6} + \frac{2K_3(a_3 - b_3P_3)}{6Q_2}$$

$$+ \frac{2c_3(a_3 - b_3P_3)}{6} + \frac{g_3c_3Q_3}{6} + \frac{K_4(a_4 - b_4P_4)}{6Q_1} + \frac{c_4(a_4 - b_4P_4)}{6} + \frac{g_4c_4Q_4}{12} - \lambda_1(Q_2 - Q_1)$$

$$- \lambda_2(Q_3 - Q_2) - \lambda_3(Q_4 - Q_3) - \lambda_4 Q_1 = 0.$$

$$\Rightarrow \frac{g_1c_1Q_1}{12} + \lambda_1 - \lambda_4 = 0.$$

$$\Rightarrow \frac{g_2c_2Q_2}{6} - \lambda_1 + \lambda_2 = 0.$$

$$\Rightarrow \frac{g_3c_3Q_3}{6} - \lambda_2 + \lambda_3 = 0.$$

$$\Rightarrow \frac{g_4c_4Q_4}{12} - \lambda_3 = 0.$$

It is known that, Q1 > 0 then in

$\lambda_1(Q_2 - Q_1) = 0, \quad \lambda_2(Q_3 - Q_2) = 0, \quad \lambda_3(Q_4 - Q_3) = 0, \quad \lambda_4 Q_1 = 0.$

$\lambda_1 Q_1 = 0$ arrive at $\lambda_4 = 0$ In a similar fashion, if $\lambda_1 = \lambda_2 = \lambda_3 = 0$, $Q_4 \leq Q_3 \leq Q_2 \leq Q_1$ does not satisfy $0 \leq Q_1 \leq Q_2 \leq Q_3 \leq Q_4$. Therefore the conclusion is $Q_2 = Q_1$, $Q_3 = Q_2$ and $Q_4 = Q_3$

i.e., $Q_1 = Q_2 = Q_3 = Q_4 = Q^*$

$$Q = \sqrt{\frac{2K_4(a_4 - b_4P_4) + 4K_3(a_3 - b_3P_3) + 4K_2(a_2 - b_2P_2) + 2K_1(a_1 - b_1P_1)}{g_1c_1 + g_2c_2 + g_3c_3 + g_4c_4}}$$

Numerical Examples

Let us consider an integrated inventory system having the following statistics with crisp parameters having following values [15], the ordering cost K=520 units, the optimal selling price P=36.52 units, unit purchasing cost c=4.75, g=0.2, constant demand rate coefficient a=100 units, price-dependent demand rate coefficient b=1.5.

As mentioned earlier, the trapezoidal numbers

$\tilde{K} = (500,515,525,540)$, $\tilde{P} = (25.28,28.55,36.18,52.00)$, $\tilde{c} = (3.95,4.10,4.35,5.05)$,

$\tilde{a} = (87,91,99,117)$, $\tilde{b} = (1.2,1.5,1.8,2.0)$, $\tilde{g} = (0.10,0.15,0.25,0.3)$

yielding the below results. Equation 1 was the optimal order quantity for crisp (equation 1) values and optimization to the EOQ is done by applying Lagrangian (equation 2) and Kuhn-Tucker (equation 3) methods under graded-mean defuzzification method. Fig. 1 represents the crisp and fuzzified output varying on demand.

Conclusion

The fuzzified output varies at a larger rate than crisp output while varying the demand. This shows that the optimization results can be obtained only by restricting the controlling parameters. Fig. 2 shows the variations in unit purchase cost for crisp and fuzzy output. An analysis of the total cost in a fuzzy environment is studied herewith by applying the Lagrangian and Kuhn-Tucker methods for optimization and using trapezoidal numbers. The value does not show much variation in optimal solutions. While varying the price-dependent demand rate coefficient among both the methods, fuzzified values decrease in Lagrangian and increases in the Kuhn-Tucker method. In Fig. 3, the curves of crisp and fuzzy EOQ remain closer. Table 2 shows the comparison of graded mean values between the two

Table 1. Comparison of Lagrangian and Kuhn Tucker method for the variation done in price dependent coefficient

Price-dependent co-efficient	Lagrangian method	Kuhn-Tucker method
b=1	EOQ = 263.6169	EOQ = 263.6169
	Fuzzy = 260.2202	Fuzzy = 321.2251
b=1.5	EOQ = 222.4949	EOQ = 222.4949
	Fuzzy = 207.7356	Fuzzy = 258.333
b=2	EOQ = 171.7967	EOQ = 171.7967
	Fuzzy = 147.1273	Fuzzy = 194.5951

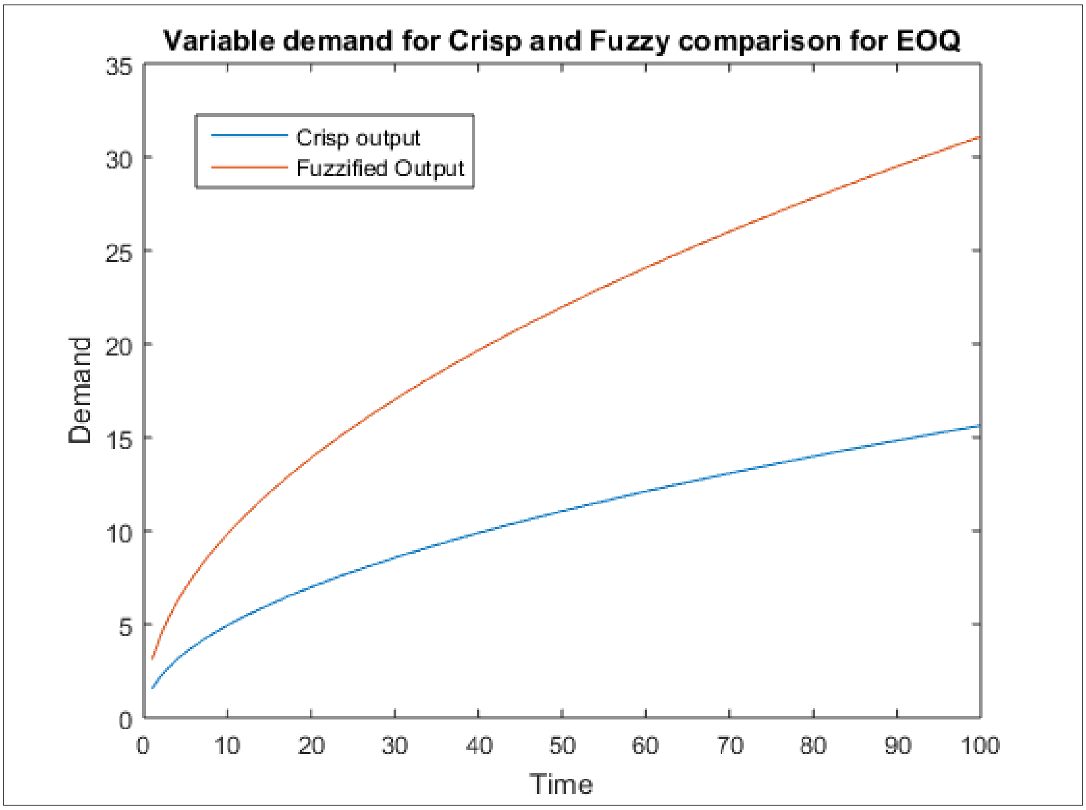

Figure 1. Variable demand for crisp and fuzzy comparison for EOQ

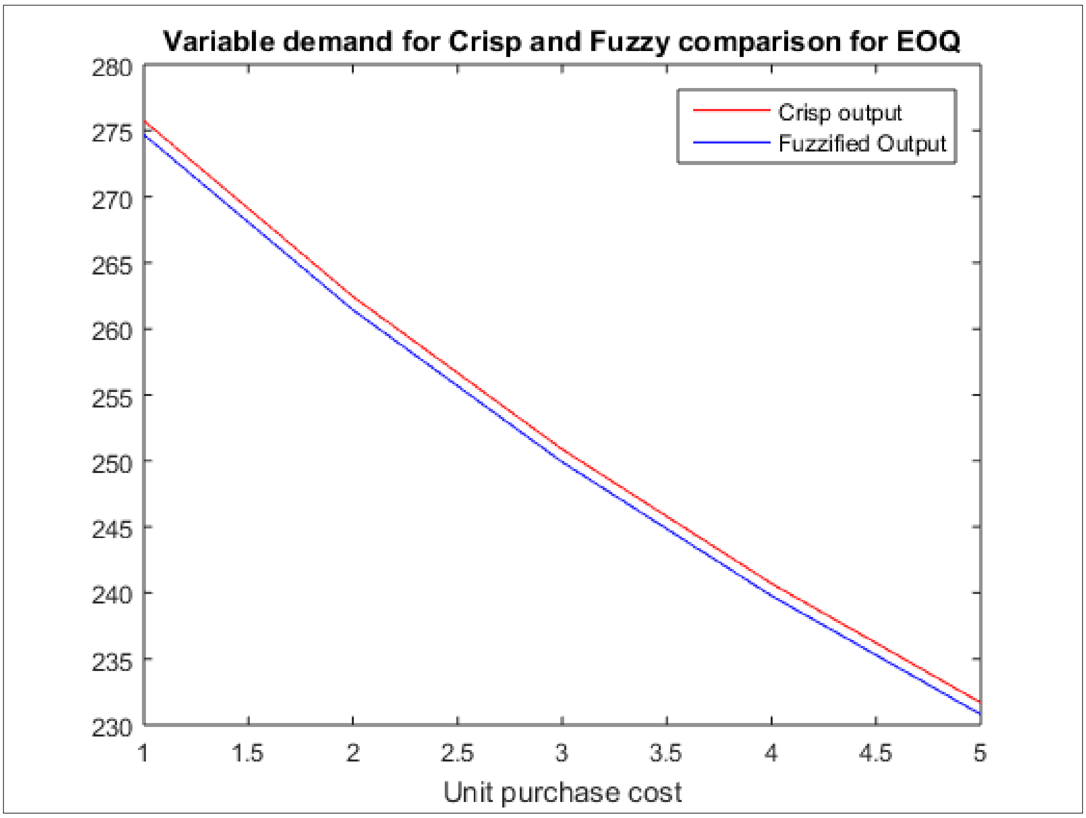

Figure 2. Variable demand for unit purchase cost for EOQ

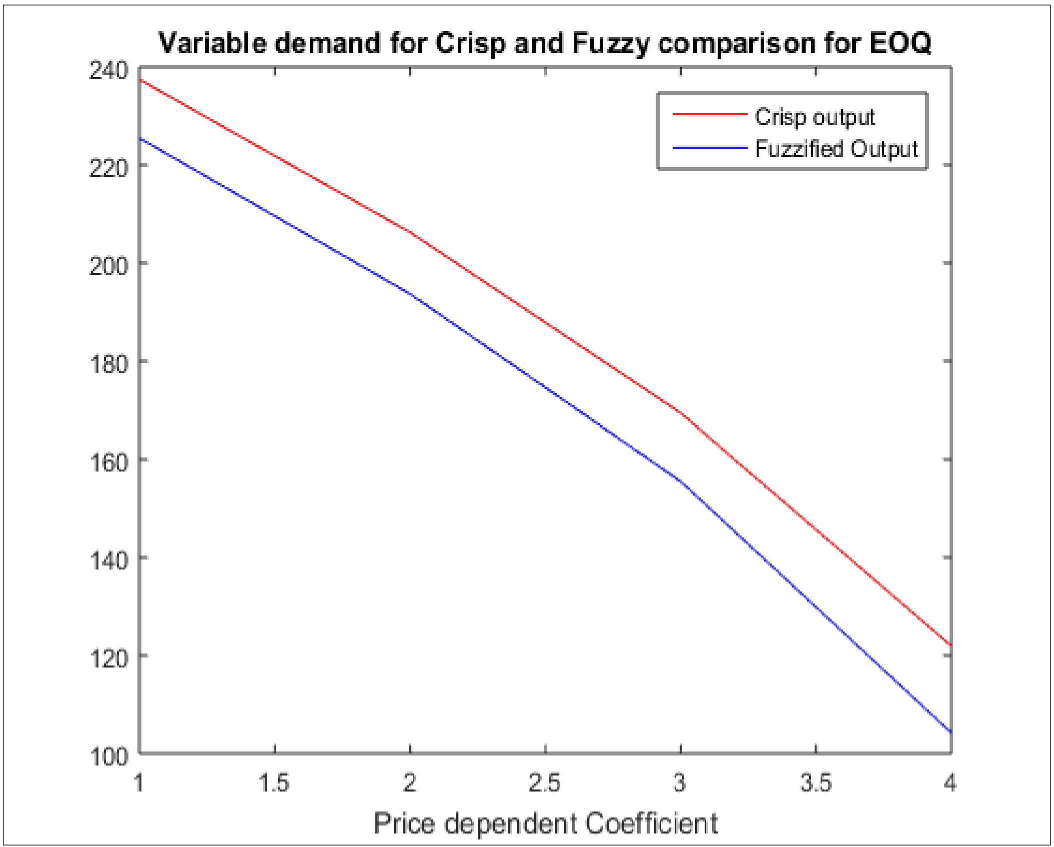

Figure 3. Variable demand for price dependent coefficient for EOQ

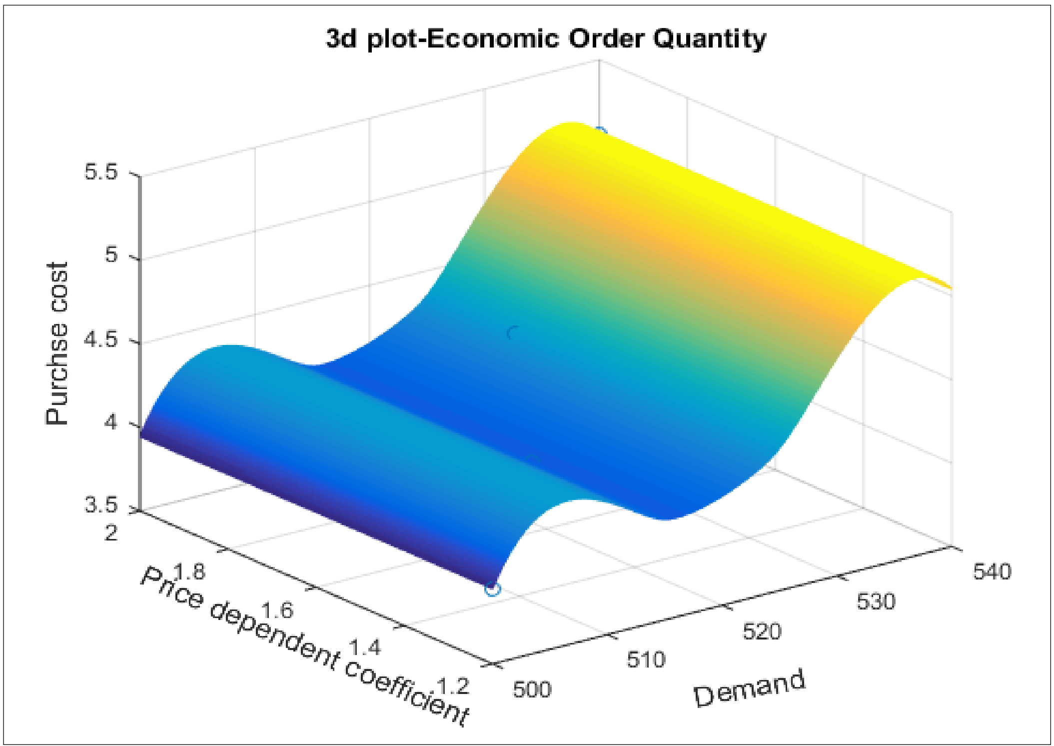

Figure 4. Three-dimensional variation of price dependent coefficient, demand and pur-

Table 2. Sensitivity analysis for Lagrangian and Kuhn Tucker method

Parameters	% change parameters	Graded-mean values	Crisp EOQ	Lagrangian Method	Kuhn-Tucker Method
K(520)	+40%	728(708,718,738,748)	263.2595	246.140	305.902
	+25%	650(630,640,660,670)	248.7569	232.451	288.931
	-25%	390(370,380,400,410)	192.6863	179.430	223.231
	-40%	312(292,302,322,332)	172.3438	160.136	199.342
P(36.52)	+40%	51.128(31.128,41.128,61.128,71.128)	159.7376	106.502	128.4246
	+25%	45.65(25.65,35.65,55.65,65.65)	185.7729	147.128	178.9012
	-25%	27.39(7.39,17.39,37.39,47.39)	253.9615	236.628	289.3353
	-40%	21.912(1.912,11.912,31.912,41.912)	271.093	257.475	315.0044
a(100)	+40%	140(120,130,150,160)	305.4398	301.807	374.9838
	+25%	125(105,115,135,145)	277.2588	271.688	338.6781
	-25%	75(55,65,85,95)	148.7803	127.422	168.3332
	-40%	60(40,50,70,80)	75.5944	52.1485	49.2549
b(1.5)	+40%	2.1(1.6,1.8,2.2,3)	159.7376	134.7605	187.338
	+25%	1.9(1.6,1.8,2,2.2)	185.7729	180.8325	227.4581
	-25%	1.1(0.5,0.8,1.4,1.7)	253.9615	250.2417	310.0121
	-40%	0.9(0.6,0.8,1,1.2)	271.0938	271.8465	334.0561
c(4.75)	+40%	6.7(6.4,6.5,6.9,7)	188.0425	169.2837	210.5158
	+25%	5.9(5.5,5.7,6,6.5)	199.0055	179.1926	222.8382
	-25%	3.6(3,3.2,3.8,4.6)	256.9150	223.8899	278.4223
	-40%	2.8(2.2,2.7,3,3.2)	287.2397	258.3873	321.3222
g(0.2)	+40%	0.28(0.18,0.2,0.3,0.5)	188.0425	170.5264	212.0612
	+25%	0.25(0.14,0.15,0.26,0.54)	199.0055	176.1179	219.0145
	-25%	0.15(0.1,0.14,0.16,0.2)	256.9150	241.4086	300.208
	-40%	0.12(0.1,0.11,0.13,0.14)	287.2397	271.2692	337.3417

optimization methods in operations research [13] Lagrangian and Kuhn-Tucker methods. The values are varied to an increase and decrease of 25% and 40% which clearly shows that the crisp values of EOQ stay perfectly aligned between both the methods. Fig. 4 collates the three parameters viz., price dependent coefficient, purchase cost, and varying demand which shows the variations in 3D model. Tab 2.

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